PHYS 320 ANALYTICAL MECHANICS

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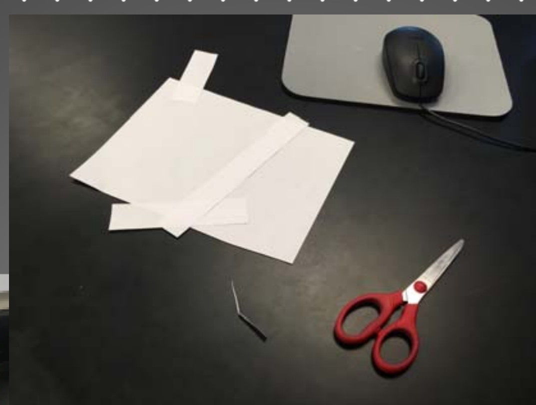
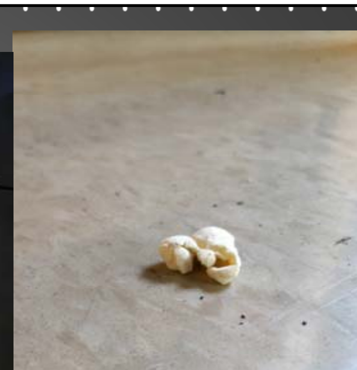
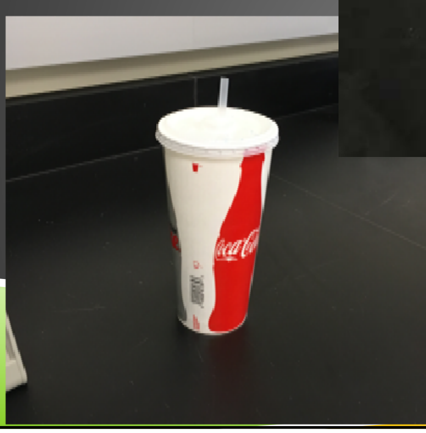
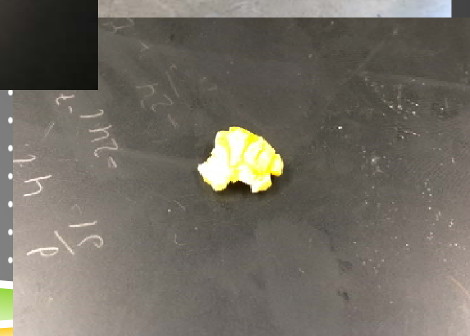
LAB . . .









Hamilton’s Principle

- *Of all the possible paths along which a dynamical system may move from one point to another within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.*

Published in two papers, 1834, 1835

Calculus of Variations

$$J = \int_{x_1}^{x_2} f(y(x), y'(x); x)\,dx$$

$$\text{where} \quad y'(x) \equiv dy/dx$$

Neighboring function: (*parametric representation*)
y(α,x) = y(0,x) + α η(x) = y(x) + α η(x)

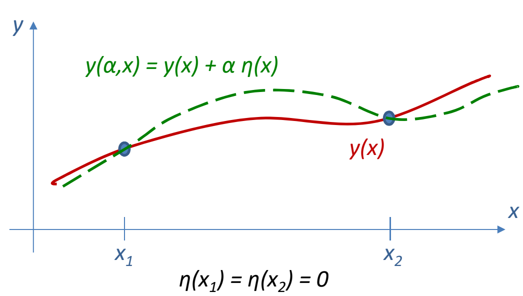
If *α = 0,* then *y(0,x) = y(0,x) = y(x)* is the function that yields the extreme value in *J*

Has extreme values (is “stationary”*) when*

$$\left.\frac{\partial J}{\partial \alpha}\right|_{\alpha=0} = 0 \quad \forall\, \eta(x)$$

$$\Rightarrow \frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

Euler-Lagrange Equation



Calculus of Variations and Hamilton's Principle

$$J = \int_{x_1}^{x_2} f(y(x), y'(x); x)\,dx$$

where $y'(x) \equiv dy/dx$

has extreme values (is "stationary") when

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

Similarly

$$S = \int_{t_1}^{t_2} L\,dt$$

where $L \equiv T - U$

Hamilton's principle

has extreme values (is stationary) when taken along the actual path, so that

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0$$

Lagrange's equation

where the q_i are generalized coordinates